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No. 160

**INSTRUMENTAL ERRORS OF A
FOUR-CHANNEL CRYSTAL-VIDEO
DIRECTION FINDER**

A. J. Jesswein

RADIO DIVISION II

23 April 1953



NAVAL RESEARCH LABORATORY, WASHINGTON, D.C.

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INSTRUMENTAL ERRORS OF A FOUR-CHANNEL
CRYSTAL-VIDEO DIRECTION FINDER

by

A. J. Jesswein

23 April 1953

Countermeasures Branch
Radio Division II
Naval Research Laboratory
Washington 25, D. C.

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CONTENTS

ABSTRACT	iv
PROBLEM STATUS	iv
AUTHORIZATION	iv
INTRODUCTION	1
BRIEF DESCRIPTION OF DIRECTION FINDER	1
FUNDAMENTAL FACTORS AFFECTING SYSTEM ERROR	2
Antenna Response Error	3
Gain Mismatch Error	4
MEASURED INSTRUMENTAL ERROR	5
COMPARISON OF MEASURED AND CALCULATED ERROR	6
System Transfer Characteristics	6
Analysis of Measured Errors	7
Discrepancies and Second-Order Effects	8
Minimum Measured Instrumental Error	9
CONSIDERATION OF CRYSTAL MATCHING AND STABILITY	9
CONCLUSIONS	11
REFERENCES	12
APPENDIX I	
Calculation of Bearing Errors, General Case	13
APPENDIX II	
Calculation of Bearing Error due to Pattern Shape Alone	14
APPENDIX III	
Calculation of Bearing Error due to Gain Mismatch Alone	16

ABSTRACT

The instrumental bearing error characteristics of a four-channel, crystal-video direction finder have been measured over the frequency range of 2800 Mc. to 6000 Mc. These measured bearing errors are presented here along with a discussion and analysis of the various first-and second-order factors that cause this error. The instrumental bearing error is found to be approximately the summation of antenna pattern shape error and gain mismatch error. The techniques utilized in this development result in an instrumental error which does not exceed ± 7.5 degrees and this is less than has been obtained previously with similar four-channel systems.

PROBLEM STATUS

This is an interim report; work is continuing on this project.

AUTHORIZATION

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INTRODUCTION

To provide an effective weapon against enemy submarines, which have become a serious potential menace, the U. S. Navy has a need for an airborne microwave direction finder capable of instantaneous interception of radar-type signals. The primary function of such a direction finder is to provide certainty of intercept against S- and X-Band radar-type signals and to present instantaneous bearings on a cathode-ray tube indicator. The conventional microwave direction finder, consisting of a rotating, directional antenna and a superheterodyne receiver, does not have a high probability of intercept against brief and intermittent radar signals under many conditions. The direction finder discussed in this report is wide-open in azimuth and frequency (over a two-to-one band at present) and does give certainty of intercept against the briefest of radar signals. All the details of this direction finder (known unofficially as the NL/ALD-A(XB-1)) are not described in this report, since it is limited to a discussion of instrumental bearing errors and the factors responsible for these errors.

BRIEF DESCRIPTION OF DIRECTION FINDER

A block diagram of the wide-open, crystal-video direction finder, given in Figure 1, indicates the basic theoretical concept of this system.

The direction finder is sometimes called a four-channel system because four complete crystal-video type receivers are employed, each fed from a separate antenna. The antenna system, at the present stage of the four-channel direction finder development, consists of four S-band horn-type antennas which are mounted on a 13-inch-diameter cylindrical ground plane having its axis vertical. (See Figures 2a and 2b). The horn elements are spaced at 90-degree intervals and oriented for acceptance of horizontally polarized signals only. Each antenna therefore receives a portion of the electromagnetic field energy available, the amount being a function of the angle of arrival of this energy. The frequency band covered by this antenna system is 2500 Mc. to 5000 Mc.

It is not intended that this report cover the many theoretical aspects involved in the antenna design. Another report is being prepared at this time containing a more detailed discussion of the design problems.

Simple $\cos \theta$ antenna pattern functions have been assumed in Figure 1, since these produce an easily identified, theoretical zero error for linear receivers. However, it should be noted that other, more complex antenna pattern functions are possible which also produce zero error, assuming perfect gain matching.

The intercepted electromagnetic energy is demodulated directly at the output of each antenna element by a crystal detector and therefore the only frequency coverage limitations of the system will be due to the antenna characteristics. This direct detection, by means of a crystal located at the antenna, is less sensitive than superheterodyne detection by approximately 30 decibels. However, it is this direct detection combined with an all-around looking antenna system which insures certainty of intercept and the actual intercept range sacrificed by using crystal-video detection is not large for an airborne system under most conditions. This was verified by a series of tests described in reference (a).

Wide-band video amplifier stages follow the crystal detector, and are designed to give optimum signal-to-noise ratio for the average width of pulses anticipated. Included as an integral part of these stages are gain controls for matching the video gains of the four receivers. Each receiver also has provisions whereby the system transfer characteristic can be made either linear or square-law by inserting or omitting a square-root-law network. When inserted, the square-rooting device counterbalances the square-law effect of the crystal detector at the antenna, thus resulting in a linear transfer function of voltage for the receiver. The four video amplifiers are identical and provide the gain necessary for a radial spot deflection on the cathode-ray tube indicator when the receiver outputs are connected to its deflection plates. The deflection angle and magnitude of the radial trace depend upon the relative vector magnitudes of coincident signal voltages applied to adjacent deflection plates; with linear amplifiers and $\cos \theta$ antenna patterns the deflection angle will be identical with the true bearing angle of the signal.

FUNDAMENTAL FACTORS AFFECTING SYSTEM ERROR

To assist in the interpretation of the measured error curves, a brief discussion of the various fundamental factors affecting the instrumental error of the system is included. This error is essentially composed of two variables, one a function of the antenna radiation pattern shape, and the other a function of the relative mismatch or gain difference between channels. Each contributes to the total system error.

Hypothetical trigonometric functions, that approximate the antenna response patterns, have been employed to calculate typical error curves for comparison with the actual measured curves. The purpose of presenting these calculated patterns and the resulting errors, under several different conditions, is to permit a better understanding of the measured instrumental error curves.

Determination of the general bearing error equation of a four-channel direction finder, as employed here, is shown in Appendix I.

Antenna Response Error

An infinite number of hypothetical antenna patterns can be drawn graphically, which result in a minimum or zero-error due to pattern shape alone (assuming matched channel responses). The majority of these, however, are not readily obtainable as actual antenna response patterns. Concurrently, the known fact that the half-power (3 db) beam-width is a function of frequency, results in pattern shape errors that are also functions of frequency and a zero-error response cannot be maintained over a two-to-one band. Therefore, it is desirable to have some indication as to the magnitude and type of error that may be encountered due to beam-width change.

An individual horn mounted with its aperture flush with the surface of a cylinder is found to have appreciable pickup beyond 180 degrees and a half-power (3 db) beam-width which ranges from 120 to 170 degrees over the two-to-one band. This variation in beam-width is for an aperture which measures 1.0 inch by 2.75 inches and would be less for a narrower aperture. To approximate the antenna pattern as described, the mathematical function,

$$f_n(\theta) = F_{on} \cos K \left[\theta + \frac{\pi}{2}(1-n) \right]$$

was assumed where K is the beam-width factor and n depends upon the relative horn position on the cylinder. (See Appendix II). Figures 3a and 3b indicate the extent to which the assumed patterns are similar to the actual measured antenna response patterns.

Instrumental error curves have been calculated using these assumed antenna patterns to illustrate the effects of pattern width upon the bearing error. These error curves, shown in Figure 4a, are based upon the assumption of matched, linear amplifiers, identical antenna response patterns and equal spacing of horn elements around the cylinder.

The octantal error seen in Figure 4a is a typical characteristic of this type of system. It will be observed whenever the radiation response pattern is a smooth, non-fluctuating function and has a magnitude at all angles over the forward 180 degree sector which is greater than that over the back 180 degree sector.

The maximum error is not very large and illustrates that antenna elements having pickup beyond 180 degrees are indeed usable in this system. Note that for some value of K between $3/5$ and $3/4$ a minimum or near-zero error results because in any given 90-degree sector the sign of the error changes from positive to negative (or vice versa) as K changes from $3/5$ to $3/4$. The calculated value of K for this approximate zero-error curve is 0.63.

Also included in Figure 4b are curves showing the error calculated using the function below:

$$E_n(\theta) = \left\{ E_{on} \cos K \left[\theta + \frac{\pi}{2}(1-n) \right] \right\}^2$$

This corresponds to the use of the same antenna response patterns of Figure 4a but combined with amplifier channels having a square-law transfer characteristic. Minimum error for this condition is seen to occur at $K = 1/2$ and K between $3/4$ and $6/7$. A decrease in maximum error is the result of using this transfer function and indicates that it would be advantageous to use a square-law system for this particular antenna response. This is not true at all times however, as will be shown later. The maximum error is less than ± 7.5 degrees in both of the previous calculations for the beam-widths indicated.

Gain Mismatch Errors

The gain of a crystal-video system is dependent upon three basic factors: the video amplifier circuitry, the antenna directivity and the conversion gain of the crystal detector, which includes minor crystal-holder effects. The conversion gain of a crystal when used for direct detection is arbitrarily defined, for this report, as the ratio of the peak video output voltage to the peak r-f input voltage. Because the crystal is non-linear, a value of conversion gain is not meaningful unless the r-f input level is specified.

The video amplifier gains of the receivers are related by reasonably constant coefficients, dependent only upon the parameters of the circuits and their variation with time. Therefore, any difference in amplifier gains is easily corrected periodically by appropriate gain controls.

The antenna gain is basically dependent upon the physical dimensions of the horn and with modern methods of fabrication, it is believed that nearly identical antenna gain functions of frequency can be attained.

The conversion gain of the detector crystal is definitely dependent upon the frequency and is the principal cause of individuality in the receiver's gain-frequency function. Because of their importance, crystal detector matching and stability will be discussed in some detail in a later section and some experimental tests of crystal matching will be described.

To indicate the typical effects of gain mismatch, a mathematical expression is assumed that again approximates the antenna response. In this case, however, it is chosen so that no octantal error is present in order to clearly illustrate only the type of error due to mismatched channels. The expression applied is of the form,

$$f_n(\theta) = F_{on} \left\{ 1 + \cos \left[\theta + \frac{\pi}{2}(n-1) \right] \right\}$$

where F_{on} is the relative coefficient of gain and n depends upon the relative

horn position on the cylinder. (See Appendix III). This function applies to only one pattern shape (i.e. one frequency) but will give curves of error due to mismatch that are typical in shape but not necessarily in magnitude of any frequency in the band. Figures 5a and 5b show the degree to which the assumed and actual antenna patterns agree.

Only a few of the many possible combinations of gain mismatch are shown in the curves of Figures 6a, b, and c but these indicate that a semi-cyclic type of error is most common. Figures 6a, b, and c have been calculated with the assumption that the system transfer characteristic is linear and 6d with the assumption that the system transfer characteristic is square-law. It has also been assumed that the mismatch in gain occurs in the r-f portion of the channels (i.e. occurs before the square-law characteristic of the crystal is encountered). Although not obvious in Figure 6, quadrantal error may occur under certain conditions of mismatch. For example, quadrantal error will appear if the fore and aft channels are mismatched, with respect to the port and starboard channels, by the same amount. The error of Figure 6c contains a small component of quadrantal error.

Comparing Figures 6a and 6d, it is seen that less mismatch error is obtained if the linear transfer characteristic is incorporated. Its final use depends upon other factors, however, as explained in a later section.

MEASURED INSTRUMENTAL ERROR

To determine the actual performance of the direction finder, several series of instrumental error curves were obtained by locating the direction finder antenna system in a clear unobstructed site about 30 feet from a transmitting (target) antenna. Arrangements were made so that the angular position of the direction finder antenna could be adjusted very accurately with respect to the target antenna. Instrumental error data was obtained by recording the angular position of the antenna and the bearing read from the d-f indicator. A series of these error curves is given in Figures 7, 8, and 9, each of which includes curves measured at four different frequencies over the operating frequency range. The conditions under which each of these curves was measured are indicated in the figures. In most of these figures the errors shown are of the types discussed earlier, that is, those resulting from the shape of the antenna response patterns and those due to gain mismatch of the channels. The instrumental errors shown in Figure 9 are for the direction finder as it is now being used. The errors are less than ± 7.5 degrees which is appreciably better than has been obtained, over a two-to-one frequency range, by workers at other Laboratories in developing similar d-f equipment. A comparison of the measured and calculated error curves is given in the following section where particular attention is given to reasons for some dissimilarities between them.

COMPARISON OF MEASURED AND CALCULATED ERROR

No absolute comparison can be made as to the magnitude of calculated and measured error, except that both are less than ± 7.5 degrees for matched channel gains. The typical octantal characteristic, due to antenna pattern shape, appears throughout in the measured error curves of Figures 7, 8, and 9. It is partially hidden in some instances, however, by a more extreme effect of mismatch in channel gains. Figures 7 and 8 illustrate the advantage of smaller errors obtained by including the antenna and crystals in matching channel gains at 3400 Mc. for a square-law system. Figures 8 and 9 compare the effects of a square-law and linear system when video (amplifier) matching is employed and the crystals are matched to ± 6 percent.

System Transfer Characteristics

As previously mentioned, the wide-open direction finder employed in these tests has provisions whereby the system transfer characteristic can be either linear or square-law, depending upon the insertion or omission of a square-root-law network. Instrumental error measurements were made under both conditions to determine their effect upon accuracy. Figures 8 and 9 and Figures 12a and 12b compare the errors involved and reveal that minimum error was produced, for this particular set of antenna responses, with a linear system. This is contrary to the calculated curves of Figure 4a and 4b of a previous section where the antenna function, $\cos K\theta$, proved to have less octantal error when a square-law system characteristic was used. It may therefore be stated that, for a minimum instrumental error, the ideal system transfer characteristic cannot be established unless the type of antenna response is known. However, with a given percent channel gain variation, a linear transfer characteristic will result in less error due to gain mismatch than will a square-law characteristic. This is seen in the calculated error curves of Figures 6a and 6d where a linear and square-law system are compared. Therefore, if with a particular group of antenna responses, the linear and square-law characteristic provide similar octantal error magnitudes, it will be beneficial to use the linear transfer characteristic so as to decrease the error due to a channel mismatch.

In general, it will be advantageous to use the lowest degree of transfer characteristic obtainable that does not increase the octantal error due to pattern shape. This may be observed by showing that the lower the degree of transfer characteristic, the smaller the change in gain at the deflection plates for a given gain variation at the crystals.

The channel gain can be expressed by:

$$G(k,p) = (kA)^p$$

where

$G(k,p)$ = channel gain

k = the gain coefficient indicating the relative gain of any channel with respect to the nominal gain

A = the nominal gain of the channel (a constant)

p = the degree of the transfer characteristic

The percent gain change, M , at the deflection plates, for any value of k , may be found by the following:

$$M = \frac{G(1,p) - G(k,p)}{G(1,p)} \times 100$$

Since $G(k,p) = (kA)^p$ and $G(1,p) = A^p$

$$M = (1 - k^p) \times 100$$

Table I illustrates what effect the transfer characteristic has upon the resulting gain variation at the deflection plates, assuming that the given gain change occurs at the crystal.

TABLE I

Gain Coefficient k	Degree of System p	Law of transfer Characteristic of System	Gain Variation at the Deflection Plates $M = (1 - k^p) \times 100$
.8	1/2	Square-root	10.5
.8	1	Linear	20.0
.8	2	Square	36.0

From this it is apparent that the least variation in gain at the deflection plates occurs for the lowest degree of transfer characteristic. Since the mismatch error depends directly upon the percent variation from nominal gain at the deflection plates, it is evident that the lowest degree transfer characteristic will provide a better matched system for a given variation in initial gain.

Analysis of Measured Errors

The total measured error of Figures 7 and 8 is a combination of octantal and semi-cyclic (or gain mismatch) error, this total approaching very nearly the summation of the two individual effects.

This summation is shown theoretically by choosing an assumed antenna response, $f_1(\theta)$, which will give rise to a small amount of octantal error, and closely approximating a response pattern, $f_2(\theta)$, which has zero octantal error. (See Figure 10a). Thus, the summation effect may be observed by comparing the mismatch error obtained from each of the two patterns of Figure 10a.

Mismatch error of the zero error function, $f_2(\theta) = \frac{1}{2}(1 + \cos \theta)$, assuming one of the channel gains is down 20 percent, is obtained directly since no octantal error is present (See Figure 6a). The mismatch error of the assumed $f_1(\theta)$ response pattern is obtained by: (1) calculating the total error for perfectly-matched channel gains (Figure 10b), (2) calculating the total error for a mismatch of 20 percent in one channel (Figure 10c), and (3) taking the difference between Figures 10b and 10c. This difference is the mismatch error. Figure 10d compares the mismatch error for the two pattern shapes, assuming a summation, and illustrates that they differ by only a small amount.

To indicate this summation under actual measured conditions, an antenna system was employed with one antenna element known to have approximately 20 percent less gain than the other three and with all crystals matched to ± 6 percent. Error data was taken under the following two conditions: (1) with only the video amplifiers matched and (2) with the complete system matched including the antenna and crystals (Figures 11a and 11b). The point-by-point difference between these curves is the error due to gain mismatch if the summation method is assumed to be true (See Figure 11c). To show that the summation is approximately true for the above conditions, the curve of Figure 11d was calculated using the mathematical function of Appendix III and choosing the relative gain coefficients to approximate the effects of antenna and crystals. The similarity of the two curves (11c and 11d) is evident and signifies that the types of mismatch error curves found by using the antenna function, $\frac{1}{2}(1 + \cos \theta)$, are typical of those actually resulting from the measured data. The calculated error curve is fairly accurate even though this function, $\frac{1}{2}(1 + \cos \theta)$, is closely similar to the antenna response at only one frequency.

Discrepancies and Second-Order Effects

Although the hypothetical trigonometric functions are similar to the actual antenna pattern responses and are used to obtain typical types of errors involved, their use is not entirely rigorous. For simplicity some minor factors affecting error have been ignored in the calculations. One such factor is that antenna response patterns are not simple, smooth functions as assumed but actually are fairly complex because of the small

aperture size, finite ground plane, and unbalanced feed. These items, especially at the lower frequencies, account for non-symmetry and second-order undulations in the resulting antenna response. These undulations occur principally at 2800 Mc. and 3400 Mc. as seen in the antenna patterns of Figure 3b. They produce added errors generally observed in the form of an erratic variation in the resulting error characteristic. This condition is readily observed in Figures 7a, 8a, and 9a.

Other second-order error effects occur when the antenna elements are not equally spaced at 90 degree intervals or the entire antenna system is not azimuthally aligned with the indicator. These are small errors generally if the relative spacing of the individual antenna elements and the orientation of the antenna assembly are within a few degrees of the correct values. The latter error is essentially a constant shifting of the error curve either up or down from the zero axis.

Minimum Measured Instrumental Error

It is interesting to note that relatively small errors occur at 4000 Mc. and 5000 Mc. in the measured error data of Figures 7, 8, and 9. This may be attributed to the fact that the antenna response patterns at these frequencies are very similar to the zero-error function, $f(\theta) = F_0 (1 + \cos \theta)$, used for mismatch error calculations. Actually the antenna pattern most nearly identical to this function lies between 4000 Mc. and 5000 Mc. and a minimum error will occur at this frequency. Because of the small error occurring at the shorter wave lengths in the S-band, the bearing error of the system at 6000 Mc. was measured to determine what magnitude of error would be obtained with a still narrower antenna response pattern. The actual antenna pattern at 6000 Mc. is shown in Figure 3b. The over-all gain of the system was matched to take into account the wide variations of crystal conversion-gain at 6000 Mc. and the resulting error curves for a linear and square-law system are shown in Figures 12a and 12b. These clearly show that the error is very small for the linear system. Wider aperture horns can therefore be employed to reduce the width of the response patterns and thereby reduce the bearing errors of the system at the low frequency end of its operating range. Such horns are presently under construction.

CONSIDERATION OF CRYSTAL MATCHING AND STABILITY

In order to study the effect of frequency upon crystal matching, the relative conversion gains of 60 1N23B crystals were measured at four uniformly distributed frequencies within the operating range of the antenna system. From this group only seven individual sets of four crystals were found that could be matched to within ± 6 percent of their respective mean value at each frequency checkpoint. A closer match than this was not possible among any four of the 60 crystals compared nor could more than four

crystals be found that would conform to the ± 6 percent match required of one set. Thus, the approximate degree of selection as well as the expected degree of mismatch are observed to be 50 percent and ± 6 percent respectively.

As determined by these tests of 60 crystals, the amount of selection required to obtain matched sets of crystals is not unreasonable; however, the effects of vibration, power overload, and temperature upon crystals have to be considered in the over-all picture. The fact is that crystals initially matched will probably not remain so for long periods of operating time except under ideal conditions. Recent flight tests, where the entire NL/ALD-A (XB-1) direction finder was installed on a P2V aircraft, have shown that after about ten flights, during a period of approximately one month, the crystal match changed from ± 6 percent to approximately ± 13 percent. However, the conversion gain versus frequency characteristics of these crystals were not seriously affected. It becomes a question, then, as to the validity of matching crystals to ± 6 percent at each frequency unless one is willing to replace crystals at frequent intervals.

A reasonable solution to this problem is to provide a means of crystal conversion gain adjustment and then require only that selected crystals have similar conversion gain versus frequency characteristics. By so doing, crystal aging effects could be partially overcome since the crystals could be matched at a given frequency at frequent intervals while installed in the equipment.

An adjustment method was employed as a part of the calibration measurements, using a variable crystal bias current for controlling the crystal conversion gain. Although this method gives a means of matching crystal conversion gains, the demodulated pulse shape is a function of the crystal bias current. Since, in this type of direction finder system, a difference in pulse shape between receiver channels introduces a looping distortion of the radial trace on the cathode-ray tube, there was some difficulty experienced in reading a bearing accurately. For this reason video amplifier gain adjustment is believed to be preferable to crystal bias gain adjustment.

The error curves of Figures 7 and 8 show the system error with and without this type of gain adjustment. Inspection of these curves indicates that since matching can be made at only one frequency, the total instrumental error may suffer at other frequencies unless the conversion gain versus frequency characteristics of all four crystals are nearly identical. It is important, then, that crystal selection be based upon obtaining identical conversion gain versus frequency characteristics.

CONCLUSIONS

It is concluded that:

(1) The instrumental bearing error of the four-channel crystal video direction finder is very nearly the summation of its two principal parts, (a) antenna pattern shape error and (b) gain mismatch error.

(2) The measured instrumental error, over a two-to-one frequency band, is within ± 7.5 degrees with the present antenna system and under the following conditions: video gains matched, transfer characteristic of receivers linear, and crystals matched to ± 6 percent. This value of instrumental error is less than has been obtained with similar four-channel systems developed at other Laboratories.

(3) The use of a linear receiver transfer characteristic, as compared to a square-law transfer characteristic, results in less bearing error due to gain mismatch, non-symmetry and pattern undulations. However, the optimum choice of the receiver transfer characteristic depends upon the shape of the antenna response patterns which also affect the bearing error.

(4) From a large group of crystals the expected degree of selection for sets of four crystals, matched to ± 6 percent, is approximately 50 percent.

(5) Although it is possible to obtain sets of crystals matched over the frequency band, these sets will not remain matched, under service conditions, for long periods of time.

(6) Unless it is possible to replace matched crystal sets at frequent intervals, some means should be provided for matching the over-all channel gains (including crystals) while the equipment is installed.

REFERENCES

- (a) Conf Ltr from NRL to BuAer Serial C-3940-44A/52 clw dated 10 April 1952, "Flight Test of the R-467 (XB-1)/ALR Crystal-Video Receiver with the AN/APA-69 Direction Finder at S- and X-Band".

APPENDIX I

Calculation of Bearing Errors, General Case

The deflection angle of the radial trace, appearing on the direction finder cathode-ray tube, is dependent upon the vector sum of simultaneous signals applied to its four deflection plates. Therefore, by letting θ be the true bearing (measured as shown in Figure 1) and $E_f(\theta)$, $E_s(\theta)$, $E_a(\theta)$, and $E_p(\theta)$ be the fore, starboard, aft, and port channel response functions respectively, the resulting d-f bearing is easily determined below.

Using complex notation, the radial d-f deflection will be given by:

$$E(\theta) = [E_f(\theta) - E_a(\theta)] + j [E_s(\theta) - E_p(\theta)]$$

$$E(\theta) = \sqrt{[E_f(\theta) - E_a(\theta)]^2 + [E_s(\theta) - E_p(\theta)]^2} \angle \phi$$

where ϕ is the deflection or d-f bearing angle and:

$$\phi = \tan^{-1} \left[\frac{E_s(\theta) - E_p(\theta)}{E_f(\theta) - E_a(\theta)} \right] \quad (1)$$

The system error can then be defined by:

$$\text{d-f error} = \phi - \theta = \tan^{-1} \left[\frac{E_s(\theta) - E_p(\theta)}{E_f(\theta) - E_a(\theta)} \right] - \theta \quad (2)$$

These are the general equations usable with any set of four antennas spaced at 90 degrees, if their response variations with θ , as well as the system transfer characteristics, are known.

APPENDIX II

Calculation of Bearing Error due to Pattern Shape Alone

The following general antenna voltage function was assumed for observation of typical pattern shape variation effects:

$$f_n(\theta) = F_{on} \cos K \left[\theta + \frac{\pi}{2} (1-n) \right] \quad (1)$$

where:

- $f_n(\theta)$ = the voltage response at any angle for the respective antennas
- F_{on} = the peak voltage magnitude of the function for the respective antennas
- n = the numerical designation of the antenna system as in Figure 1, so that the Fore, Starboard, Aft, and Port antennas are 1, 2, 3, and 4 respectively
- K = the beam-width factor
- θ = the true bearing angle

The voltage response at the indicator for a linear system will then be:

$$E_n(\theta) = A_n f_n(\theta) \quad (2)$$

where

- $E_n(\theta)$ = the indicator response voltage of channel n
- A_n = the relative voltage gain of channel n

For the antenna system designation of Figure 1, and letting $A_n F_{on} = E_{on}$, the indicator response functions of θ are:

$$\begin{aligned} E_F = E_1(\theta) &= E_{01} \cos K \theta & -\pi/2 \leq K\theta \leq \pi/2 \\ E_S = E_2(\theta) &= E_{02} \cos K(\theta - \pi/2) & -\pi/2 \leq K(\theta - \pi/2) \leq \pi/2 \\ E_A = E_3(\theta) &= E_{03} \cos K(\theta - \pi) & -\pi/2 \leq K(\theta - \pi) \leq \pi/2 \\ E_P = E_4(\theta) &= E_{04} \cos K(\theta - 3\pi/2) & -\pi/2 \leq K(\theta - 3\pi/2) \leq \pi/2 \end{aligned}$$

Using the general equation for d-f bearing angle, equation (1), Appendix I, and substituting the above conditions, letting $E_{01} = E_{02} = E_{03} = E_{04}$, the d-f bearing angle

$$\phi = \tan^{-1} \left[\frac{\cos K(\theta - \pi/2) - \cos K(\theta - 3\pi/2)}{\cos K\theta - \cos K(\theta - \pi)} \right] \quad (3)$$

The coefficient K effectively alters the angular span of the assumed response and therefore is likened to the beam-width variation in an antenna response due to frequency. Thus, with a given value of K, the error may be calculated for any value of bearing angle.

APPENDIX III

Calculation of Bearing Error due to Gain Mismatch Alone

The same general equations of d-f bearing angle and error are used here as in Appendix I.

The general antenna function of voltage, assumed for indicating the effects of gain mismatch, gives zero error for equal response of four antennas. The assumed function is:

$$f_n(\theta) = F_{on} \left\{ 1 + \cos \left[\theta - \pi/2 (n-1) \right] \right\} \quad (1)$$

where

- $f_n(\theta)$ = the voltage response at any angle for the respective antennas
- n = the numerical designation of the antenna system as in Figure 1, so that the Fore, Starboard, Aft, and Port antennas are 1, 2, 3, and 4 respectively
- F_{on} = the peak voltage magnitude of the function for the respective antennas
- θ = the true bearing angle

The voltage response at the indicator for a linear system will then be:

$$E_n(\theta) = A_n f_n(\theta) \quad (2)$$

where

- $E_n(\theta)$ = the indicator response voltage of channel n
- A_n = the relative voltage gain of channel n

Using the antenna system designation of Figure 1, and letting $A_n F_{on} = E_{on}$, the indicator response functions of θ become:

$$\begin{aligned} E_F &= E_1(\theta) = E_{01} (1 + \cos \theta) & \text{where } E_{01} &= \frac{E_1(0)}{2} \\ E_S &= E_2(\theta) = E_{02} (1 + \sin \theta) & E_{02} &= \frac{E_2(90)}{2} \\ E_A &= E_3(\theta) = E_{03} (1 - \cos \theta) & E_{03} &= \frac{E_3(180)}{2} \\ E_P &= E_4(\theta) = E_{04} (1 - \sin \theta) & E_{04} &= \frac{E_4(270)}{2} \end{aligned}$$

Using the general expression for d-f bearing angle, equation (1), Appendix I, and substituting the above conditions then:

$$\phi = \tan^{-1} \left[\frac{E_{02} (1 + \sin \theta) - E_{04} (1 - \sin \theta)}{E_{01} (1 + \cos \theta) - E_{03} (1 - \cos \theta)} \right] \quad (3)$$

$$\phi = \tan^{-1} \left[\frac{(E_{02} + E_{04}) \sin \theta + (E_{02} - E_{04})}{(E_{01} + E_{03}) \cos \theta + (E_{01} - E_{03})} \right]$$

It is easily seen that for $E_{01} = E_{02} = E_{03} = E_{04}$ the error at all angles is zero since:

$$\tan \phi = \tan \theta$$

and

$$\phi = \theta$$

This shows that it is possible to use equation 1 to calculate the typical error due to gain mismatch alone.

ILLUSTRATIONS

FOR

NRL MEMORANDUM REPORT NO. 160

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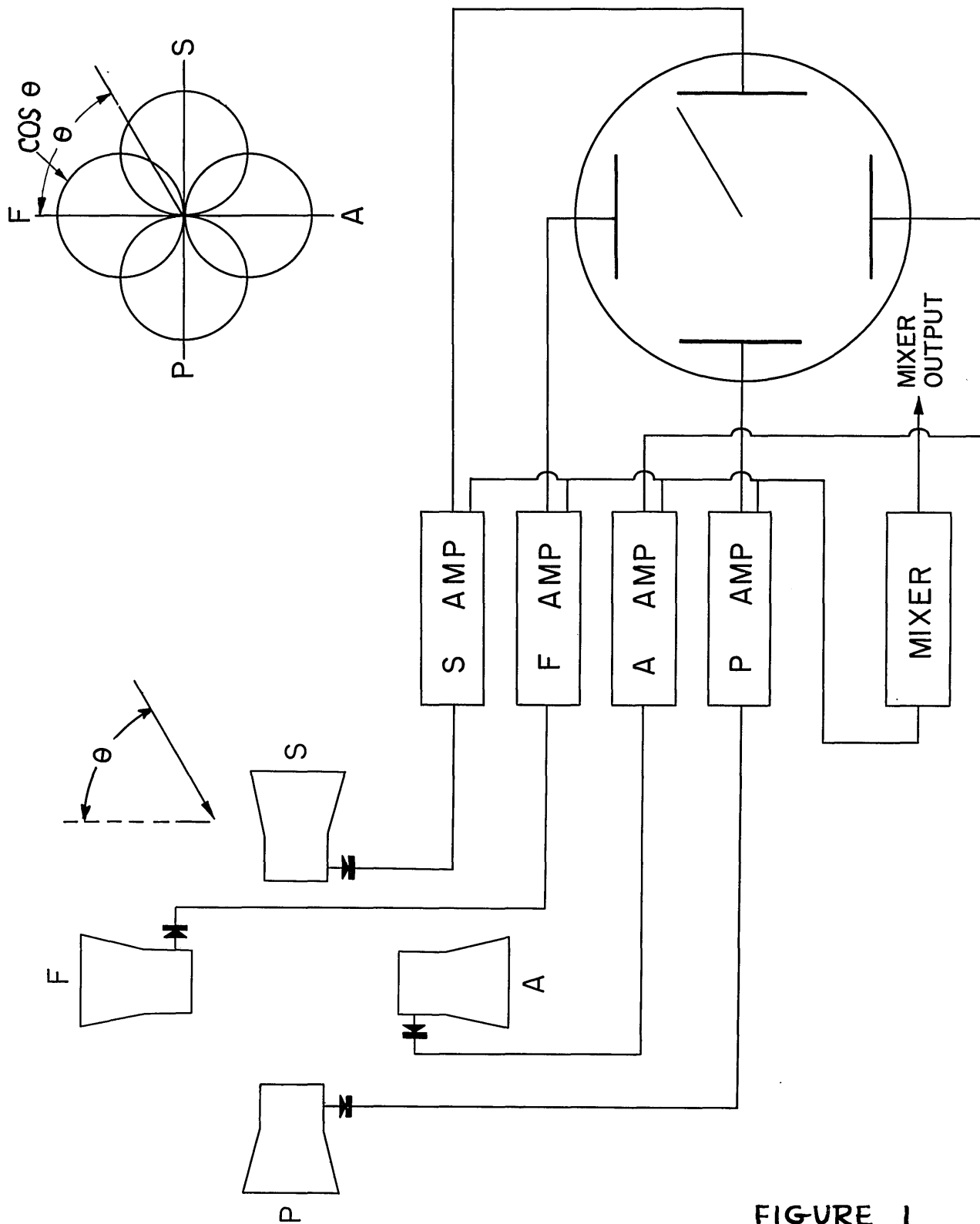
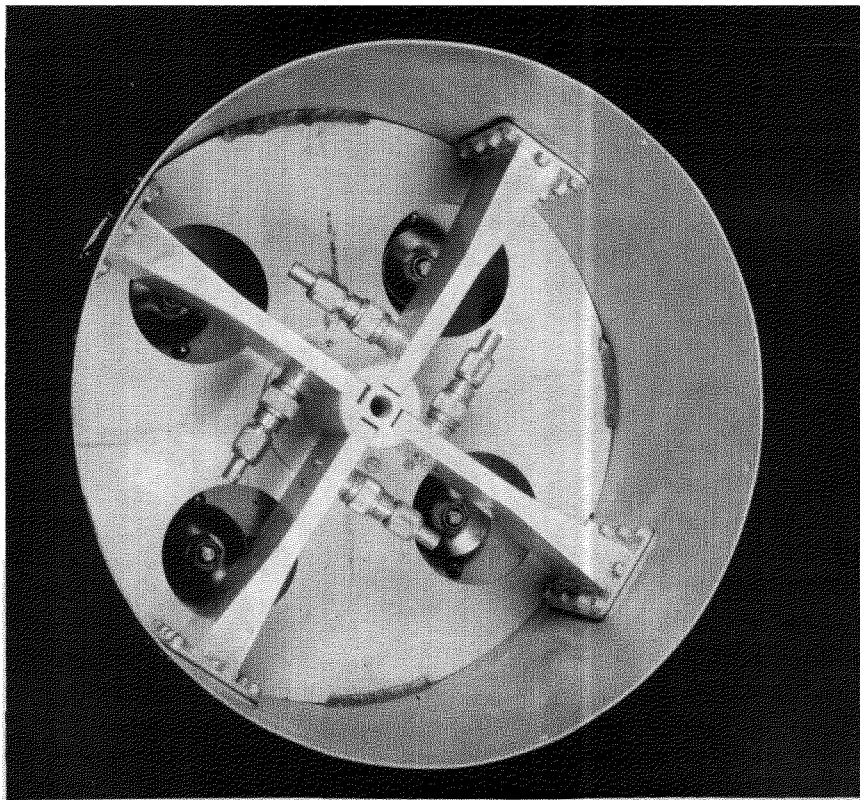
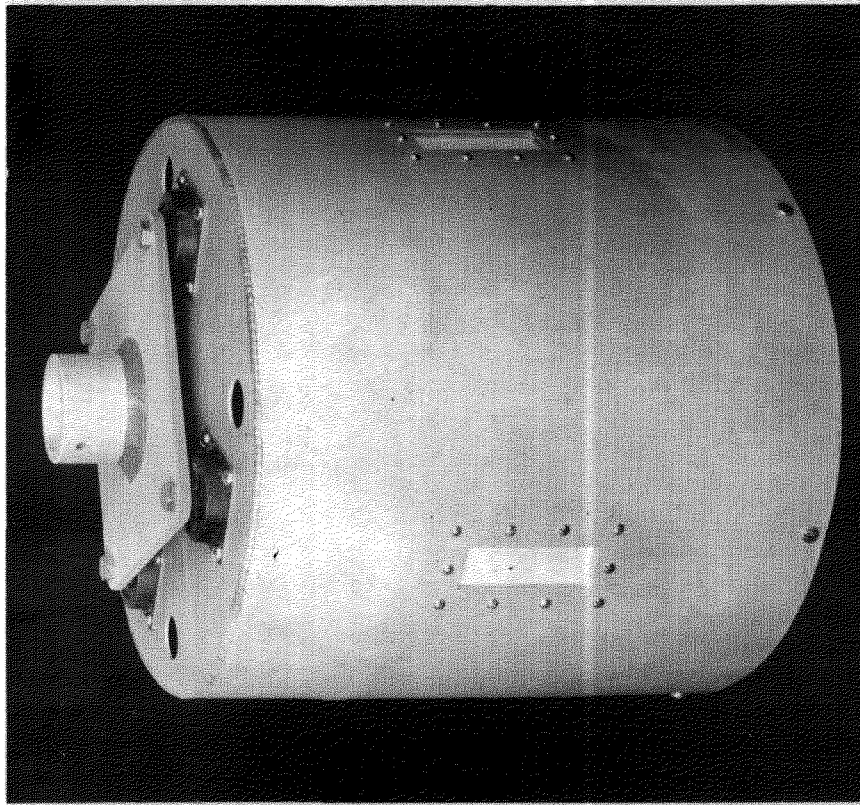


FIGURE 1

BLOCK DIAGRAM OF 4-CHANNEL CRYSTAL-VIDEO DIRECTION FINDER



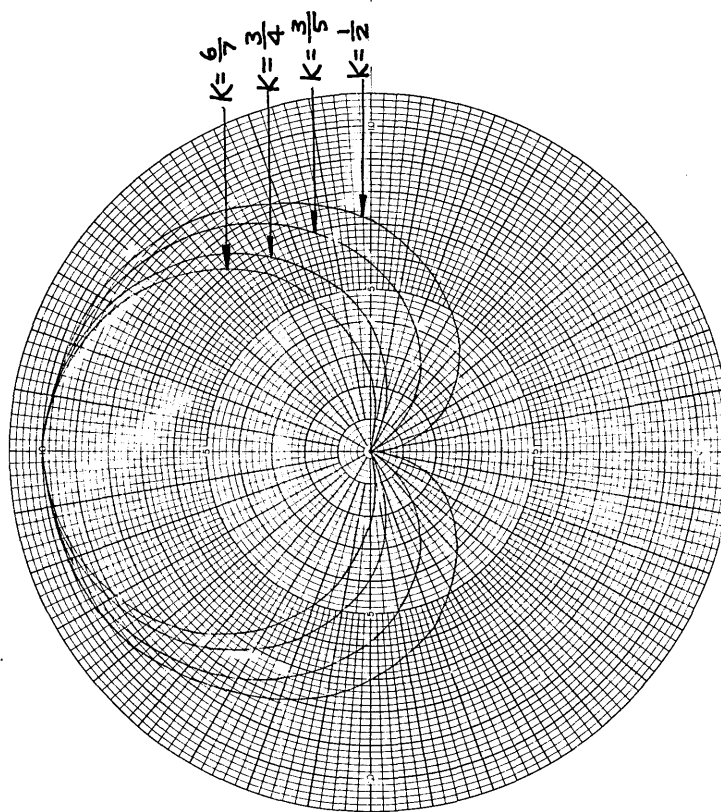
(a) INSIDE VIEW, BASE REMOVED



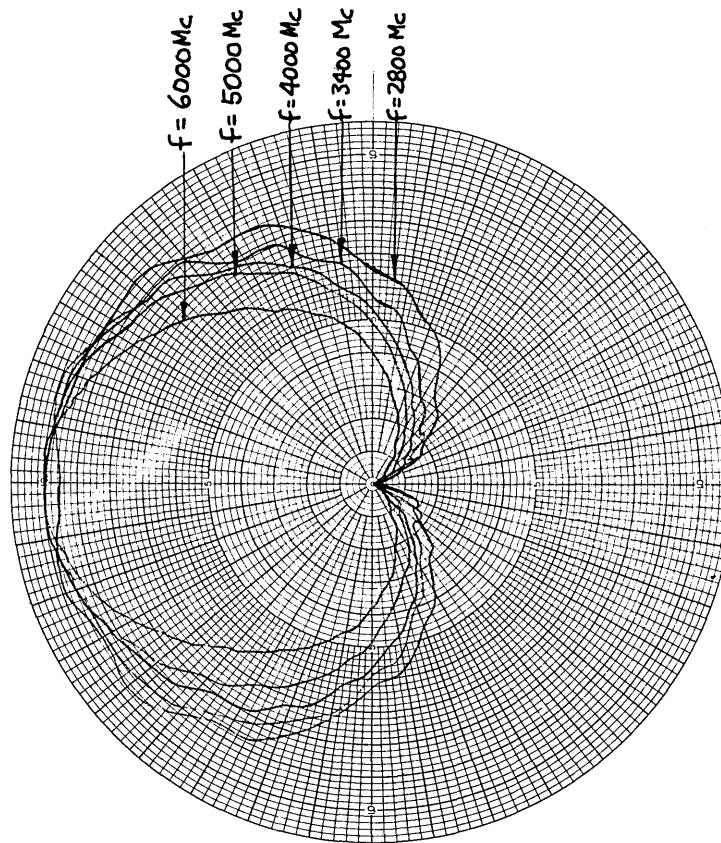
(b) OUTSIDE VIEW

ANTENNA ASSEMBLY

(a.)
CALCULATED ANTENNA PATTERNS
FOR $f(\theta) = \cos k\theta$

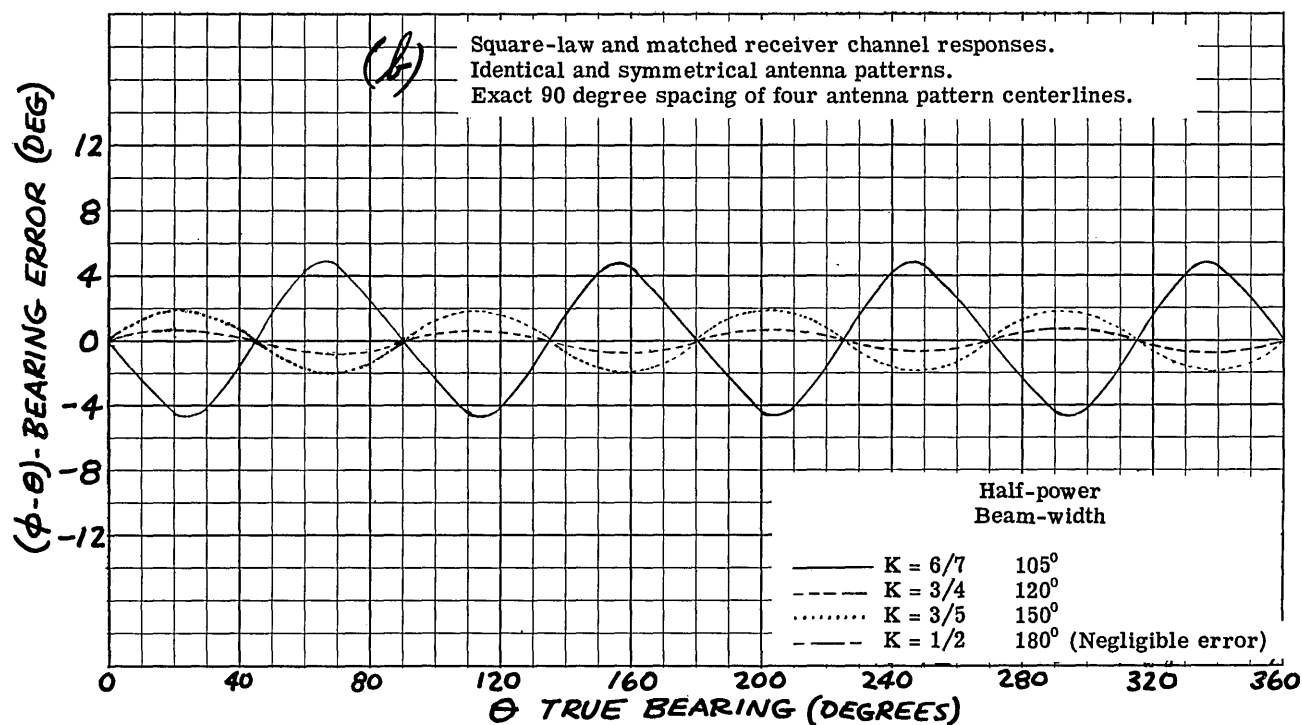
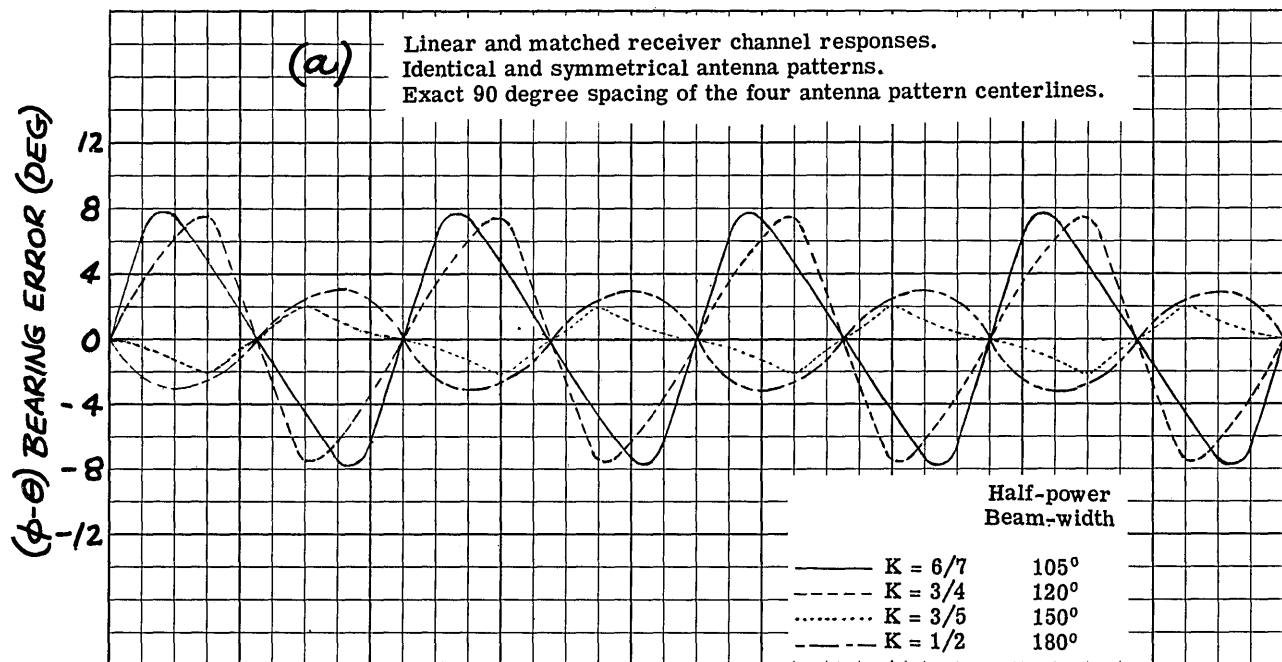


(b.)
ACTUAL ANTENNA RESPONSE PATTERN OF A
HORN ELEMENT MOUNTED ON A CYLINDER



CALCULATED AND ACTUAL ANTENNA PATTERNS

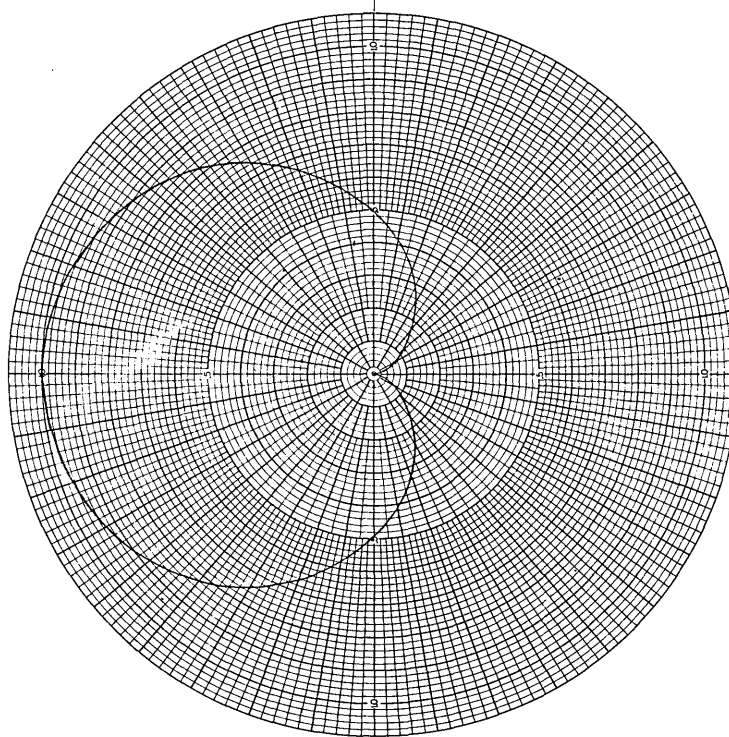
FIGURE 3



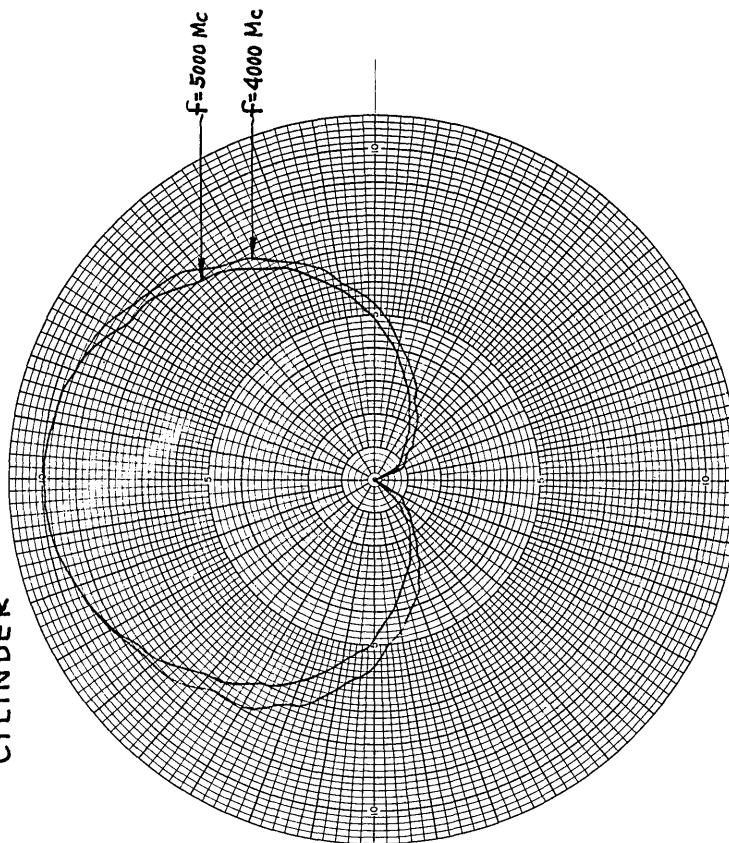
CALCULATED ERROR CURVES FOR $\cos K\theta$ TYPE ANTENNA PATTERNS

FIGURE 4

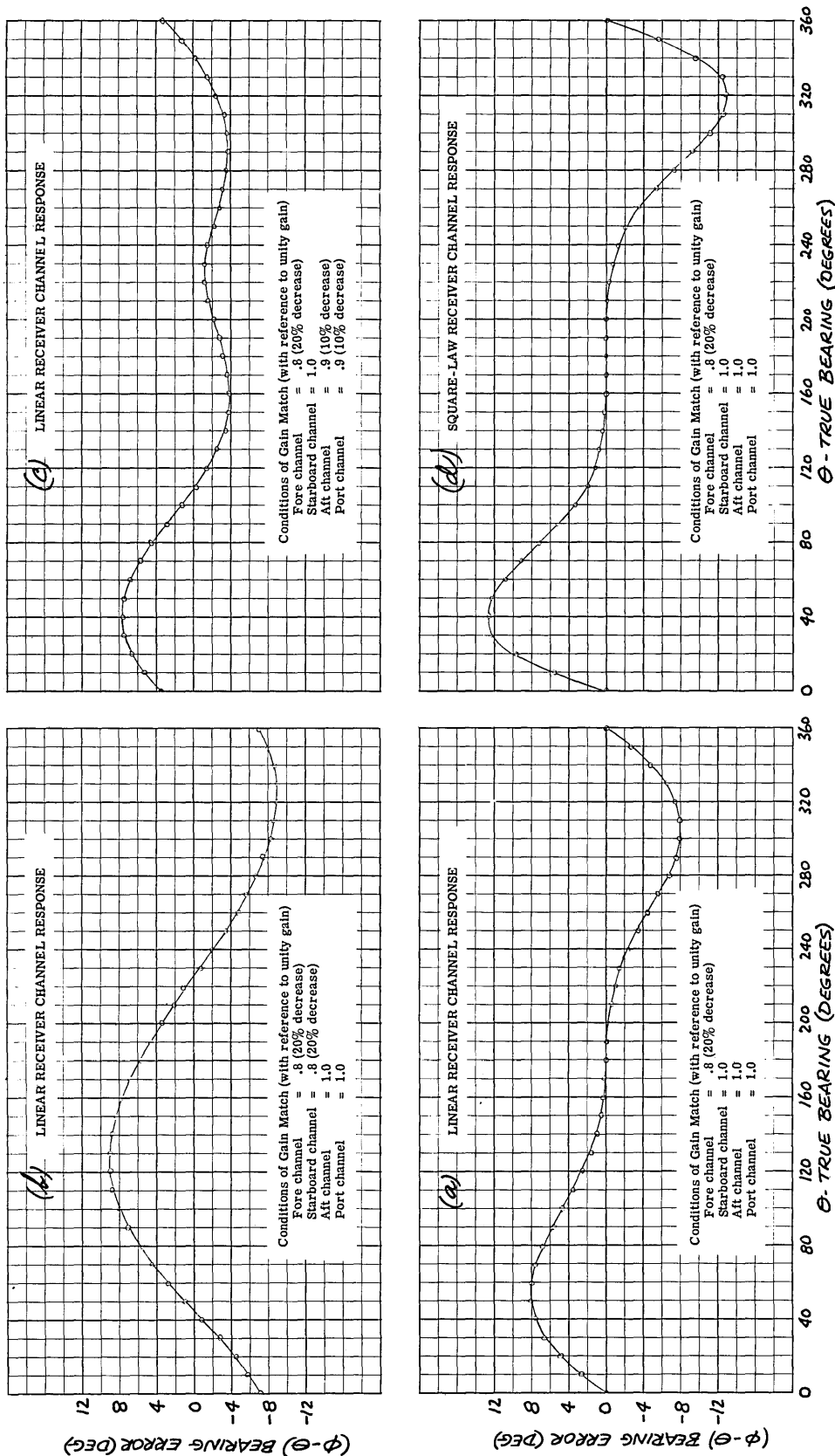
(a)
CALCULATED ANTENNA PATTERN
FOR $f(\theta) = \frac{1}{2}(1 + \cos \theta)$



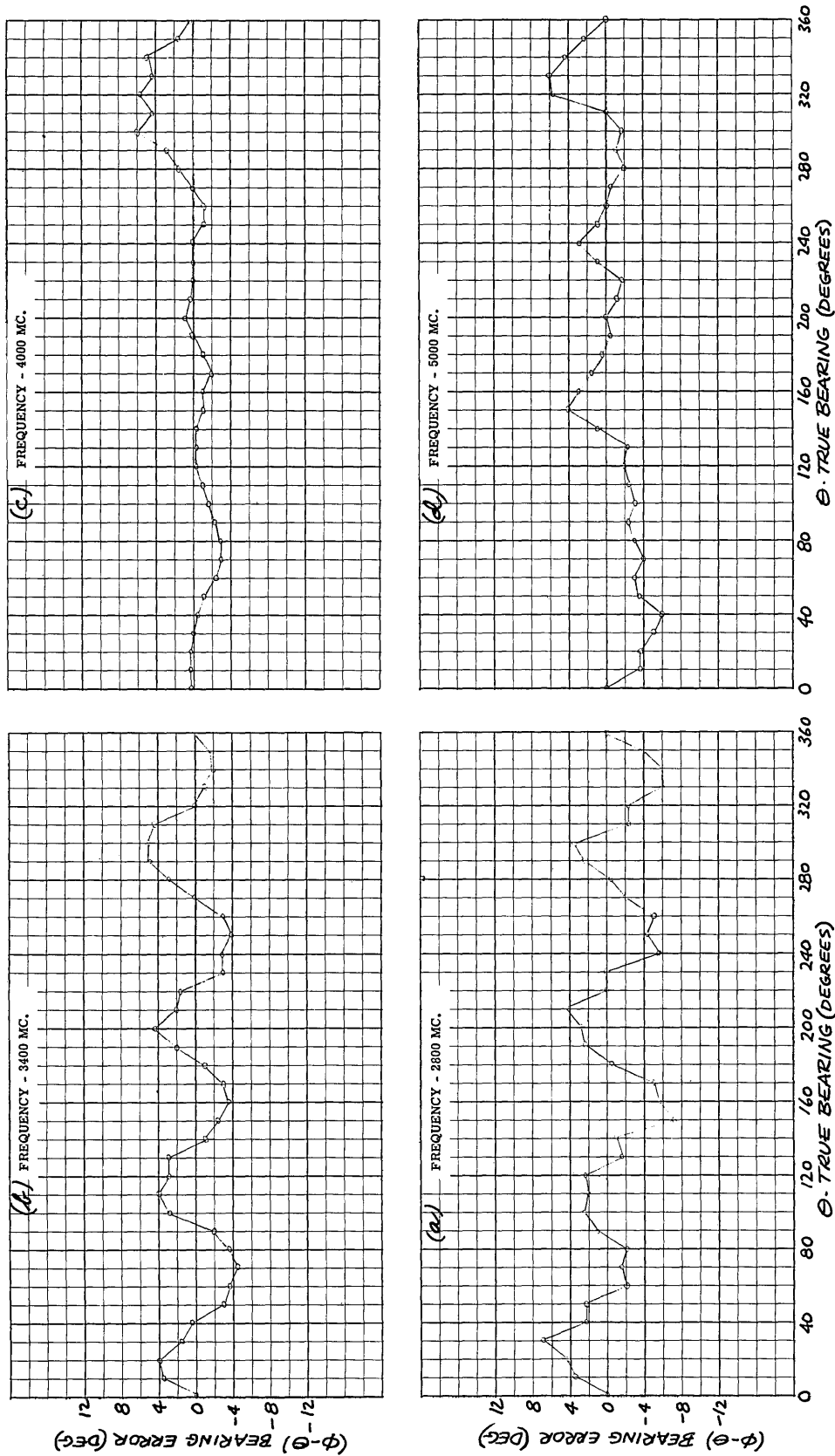
(b)
ACTUAL ANTENNA RESPONSE PATTERN
OF A HORN ELEMENT MOUNTED ON A
CYLINDER



CALCULATED AND ACTUAL ANTENNA RESPONSE PATTERNS



CALCULATED GAIN MISMATCH ERROR CURVES FOR $f(\theta) = A(1 + \cos\theta)$ TYPE ANTENNA PATTERNS, CONDITIONS ARE: IDENTICAL AND SYMMETRICAL ANTENNA PATTERNS - EXACT 90 DEGREE SPACING OF THE FOUR ANTENNA PATTERNS



MEASURED ERROR CURVES OF THE 4-CHANNEL D-F SYSTEM

CONDITIONS ARE: SQUARE-LAW SYSTEM RESPONSE, CRYSTAL MATCH $\pm 6\%$,
CHANNEL GAINS MATCHED FOR RF AT 3400 MC BY CRYSTAL BIAS CURRENT
(INCLUDES ANTENNA AND CRYSTALS)

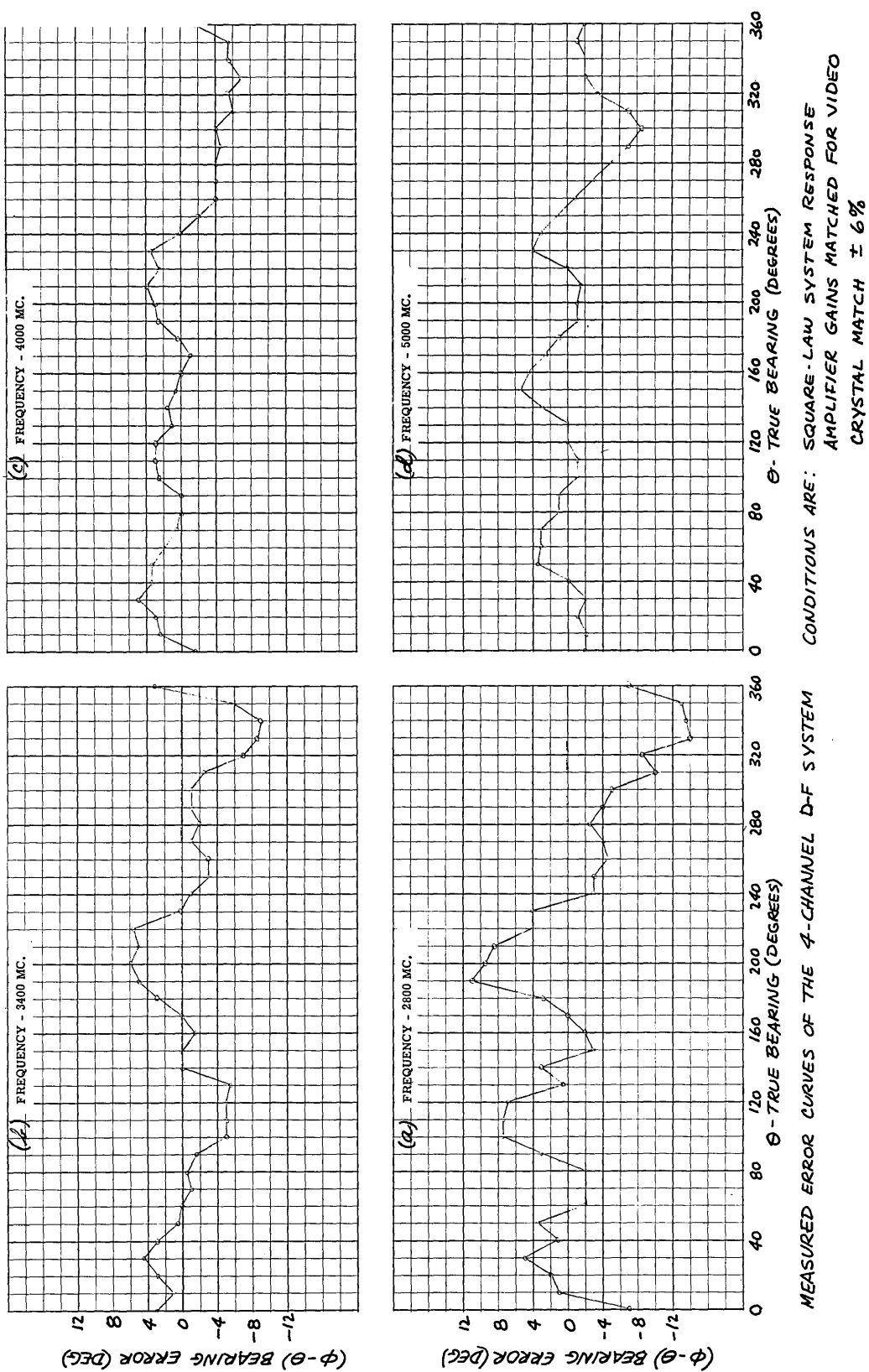
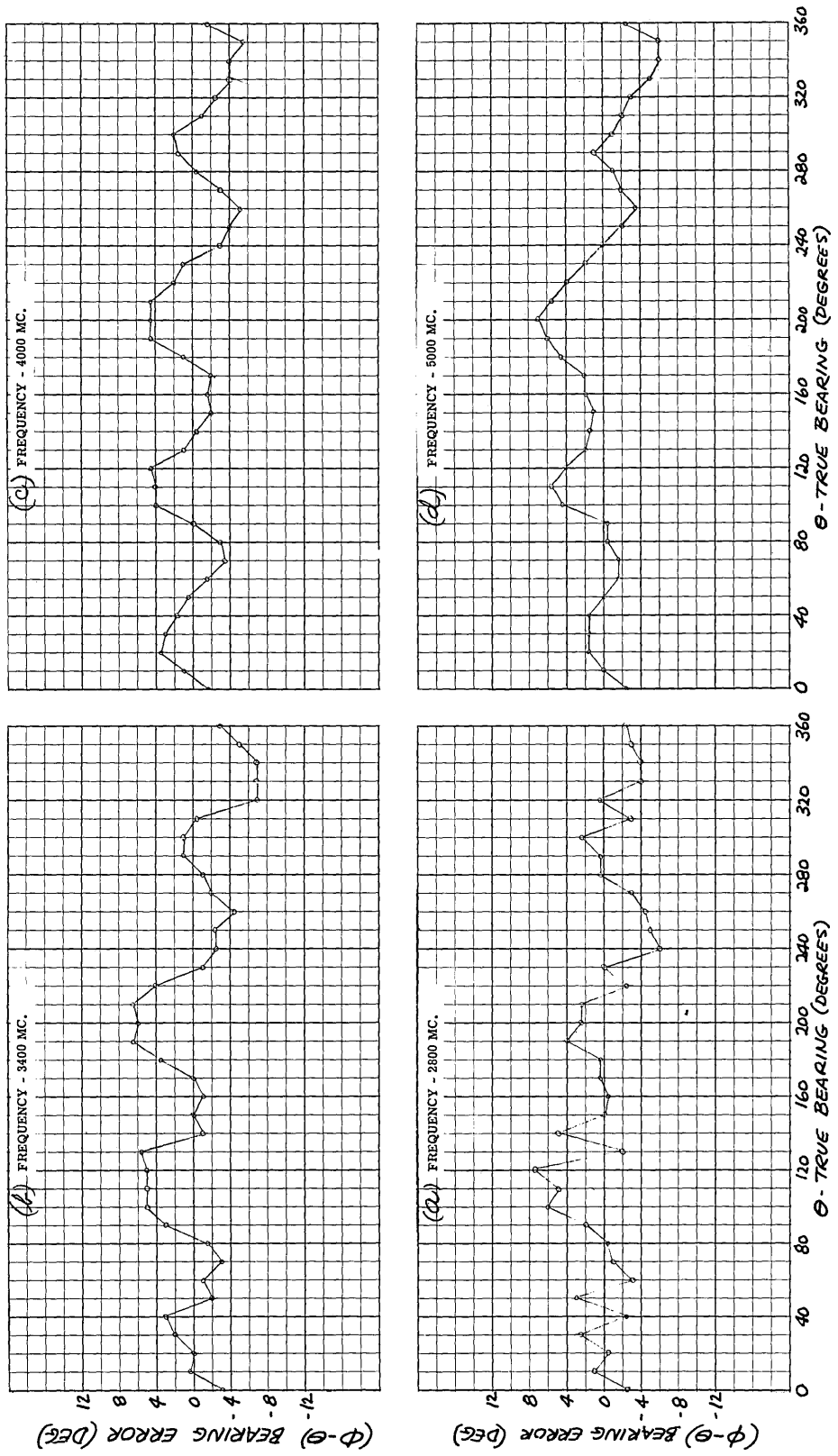


FIGURE 8



MEASURED ERROR CURVES OF THE 4-CHANNEL D-F SYSTEM CONDITIONS ARE: LINEAR SYSTEM RESPONSE
 AMPLIFIER GAINS MATCHED FOR VIDEO
 CRYSTAL MATCH $\pm 6\%$

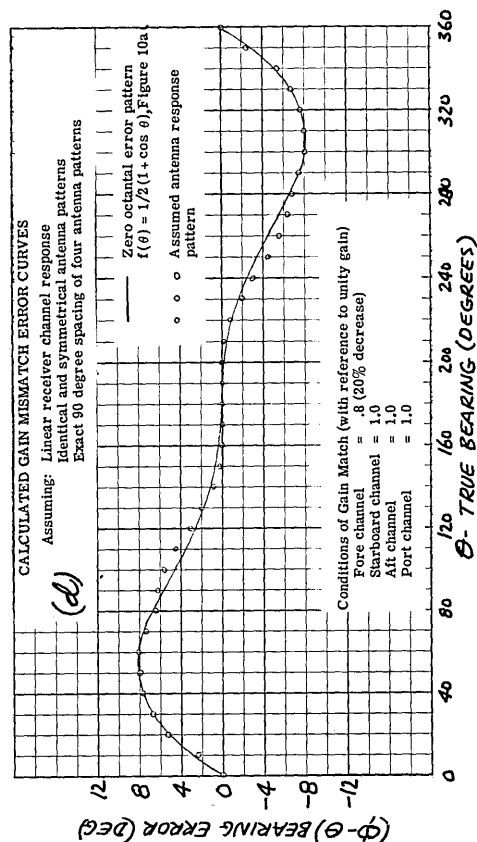
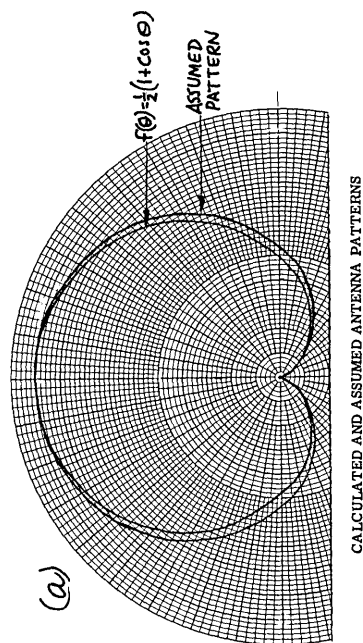
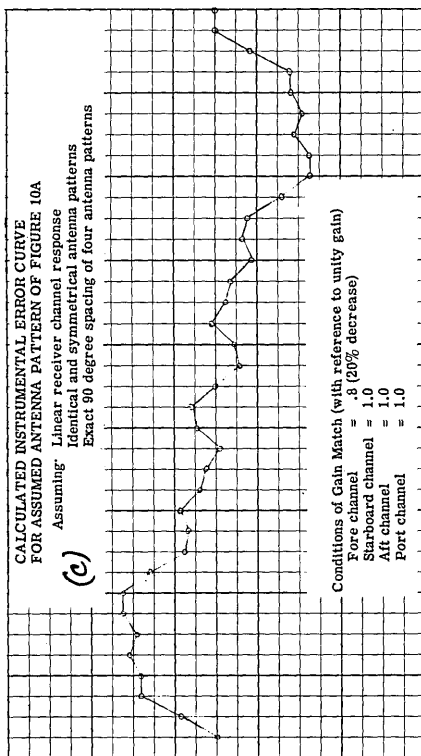
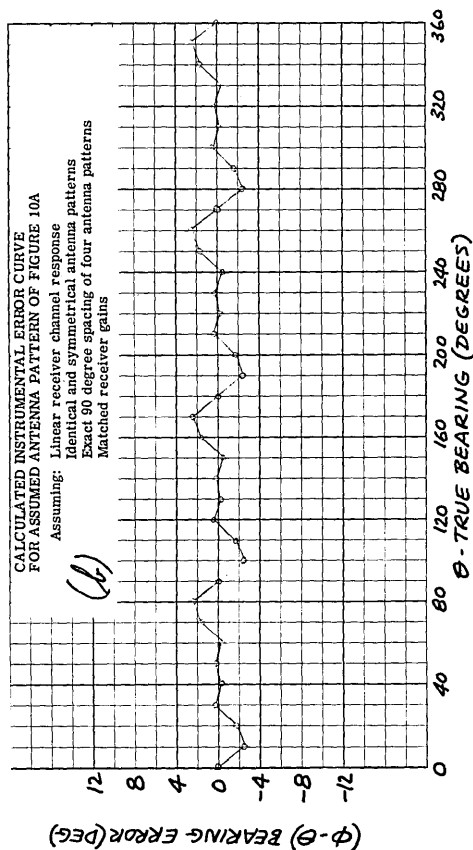
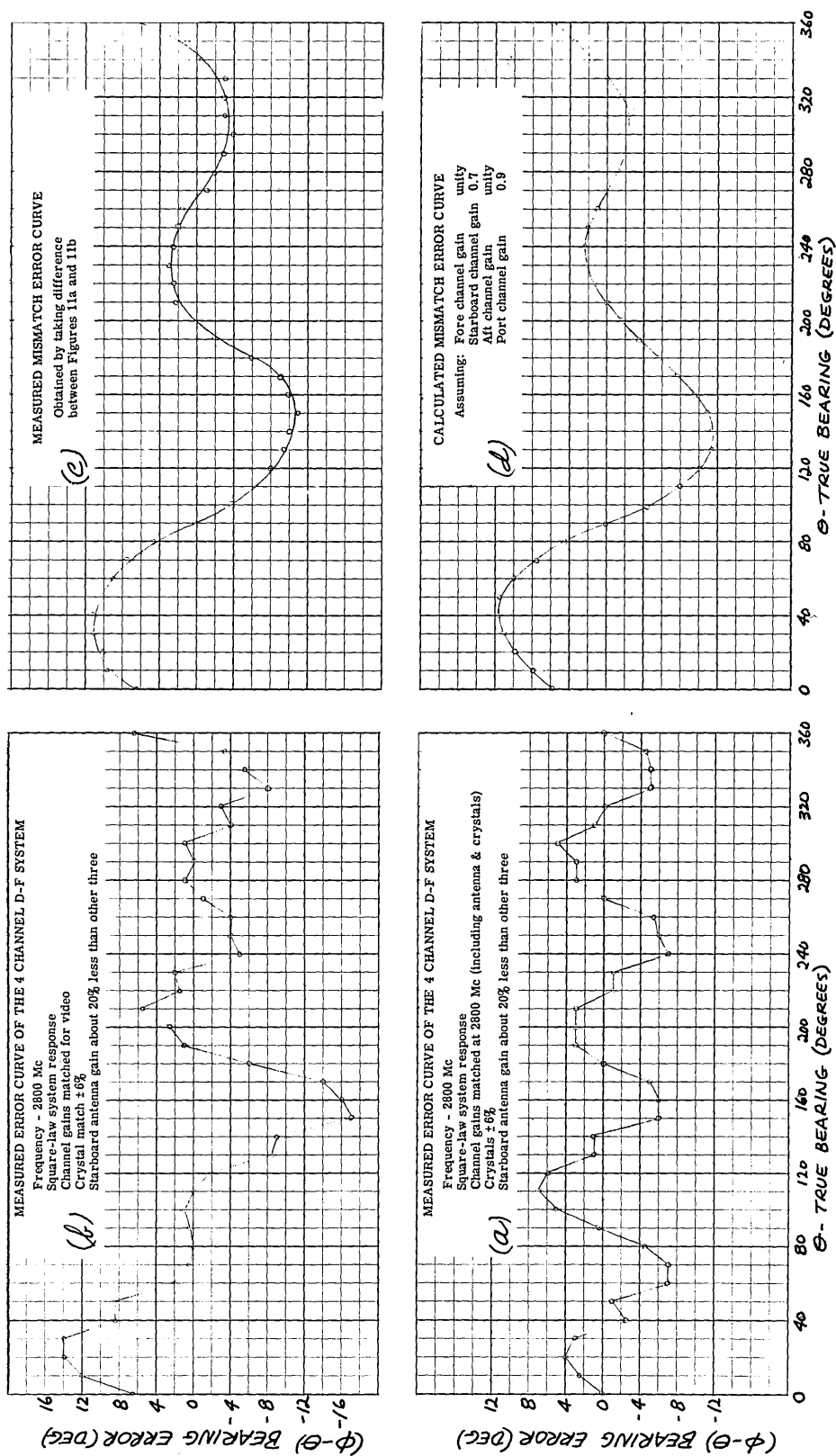
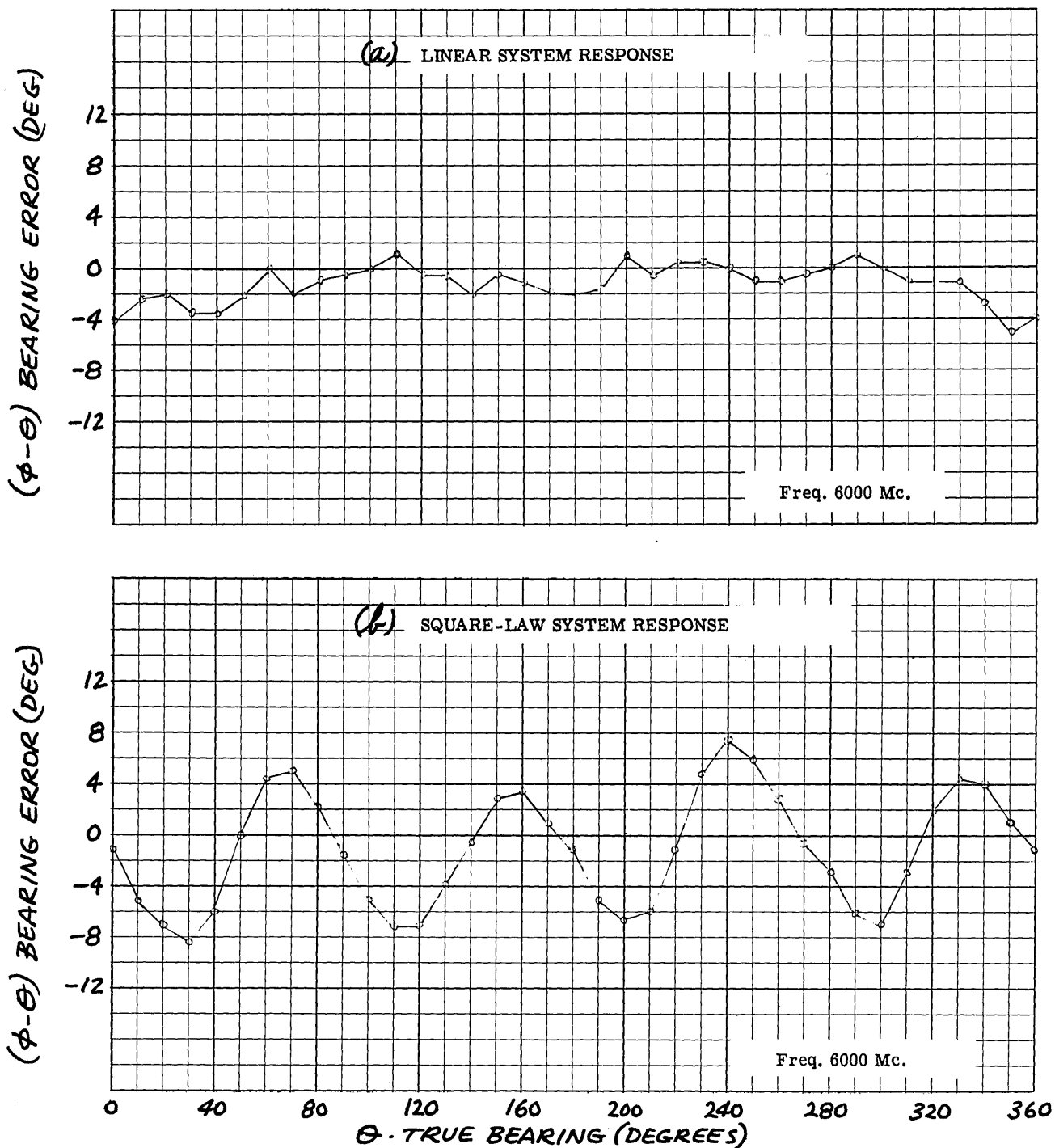


FIGURE 10



COMPARISON OF CALCULATED AND MEASURED MISMATCH ERROR



MEASURED ERROR CURVES OF THE 4-CHANNEL D-F SYSTEM
(CHANNEL GAINS MATCHED FOR RF AT 6000 Mc)

FIGURE 12